Parallel Fast Fourier Transform

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Fourier Transform

\[ H(f) = \int_{-\infty}^{\infty} h(t)e^{2\pi ift} \, dt \]

\[ h(t) = \int_{-\infty}^{\infty} H(f)e^{-2\pi ift} \, df \]

Frequency Domain \quad \longleftrightarrow \quad Time Domain

Real Space \quad \longleftrightarrow \quad Reciprocal Space
Properties

**Convolution Theorem**

\[ \int_{-\infty}^{+\infty} g(\tau)h(t-\tau)\,d\tau \Leftrightarrow G(f)H(f) \]

**Correlation Theorem**

\[ \text{Corr}(h,h) \equiv \int_{-\infty}^{+\infty} h(\tau)h(t+\tau)\,d\tau \Leftrightarrow |H(f)|^2 \]

**Power Spectrum**

\[ \text{Corr}(h,h) \equiv \int_{-\infty}^{+\infty} h(\tau)h(t+\tau)\,d\tau \Leftrightarrow |H(f)|^2 \]
Discrete Fourier Transform (DFT)

In many application contexts the Fourier transform is approximated with a Discrete Fourier Transform (DFT):

\[
H(f_n) = \int_{-\infty}^{\infty} h(t) e^{2\pi i f_n t} \, dt \approx \sum_{k=0}^{N-1} h_k e^{2\pi i f_n t_k} \Delta = \Delta \sum_{k=0}^{N-1} h_k e^{2\pi i f_n t_k}
\]

\[
\begin{align*}
    t_k &= \Delta k / N \\
    f_n &= n / \Delta
\end{align*}
\]

\[
H(f_n) = \Delta \sum_{k=0}^{N-1} h_k e^{2\pi i k n / N}
\]

The last expression is periodic, with period N. It define a between 2 sets of numbers, \( H_n \) & \( h_k \) ( \( H(f_n) = \Delta H_n \) )
Discrete Fourier Transform (DFT)

\[ H_n = \sum_{k=0}^{N-1} h_k e^{2\pi i kn/N} \]
\[ h_k = \frac{1}{N} \sum_{n=0}^{N-1} H_n e^{-2\pi i kn/N} \]

frequencies from 0 to fc (maximum frequency) are mapped in the values with index from 0 to N/2-1, while negative ones are up to -fc mapped with index values of N / 2 to N

Scale like N*N
The DFT can be calculated very efficiently using the algorithm known as the FFT, which uses symmetry properties of the DFT:

\[
F_k = \sum_{j=0}^{N-1} e^{2\pi i jk/N} f_j
\]

\[
= \sum_{j=0}^{N/2-1} e^{2\pi i k(2j)/N} f_{2j} + \sum_{j=0}^{N/2-1} e^{2\pi i k(2j+1)/N} f_{2j+1}
\]

\[
= \sum_{j=0}^{N/2-1} e^{2\pi i k j/(N/2)} f_{2j} + W^k \sum_{j=0}^{N/2-1} e^{2\pi i k j/(N/2)} f_{2j+1}
\]

\[
= F_k^e + W^k F_k^o
\]
Fast Fourier Transform

\[ F_k = F_k^e + W^k F_k^o \]

- DFT of even terms
- DFT of odd terms
- \( \exp(2\pi i/N) \)
Fast Fourier Transform

**Now Iterate:**

\[ F_e = F_{ee} + W^{k/2} F_{eo} \]

\[ F_o = F_{oe} + W^{k/2} F_{oo} \]

---

*You obtain a series for each value of \( f_n \)*

\[ F_{oeoeooeo..oe} = f_n \]

---

Scale like \( N \times \log N \) (binary tree)
Introduction

FFTW is a C subroutine library for computing the Discrete Fourier Transform (DFT) in one or more dimensions, of both real and complex data, and of arbitrary input size. We believe that FFTW, which is free software, should become the FFT library of choice for most applications. Our benchmarks, performed on on a variety of platforms, show that FFTW's performance is typically superior to that of other publicly available FFT software. Moreover, FFTW's performance is portable: the program will perform well on most architectures without modification.

It is difficult to summarize in a few words all the complexities that arise when testing many programs, and there is no "best" or "fastest" program. However, FFTW appears to be the fastest program most of the time for in-order transforms, especially in the multi-dimensional and real-complex cases (Kasparov is the best chess player in the world even though he loses some games). Hence the name, "FFTW," which stands for the somewhat whimsical title of "Fastest Fourier Transform in the West." Please visit the benchFFT home page for a more extensive survey of the results.

The FFTW package was developed at MIT by Matteo Frigo and Steven G. Johnson.
• Written in C
• Fortran wrapper is also provided
• FFTW adapt itself to your machines, your cache, the size of your memory, the number of register, etc...
• FFTW doesn't use a fixed algorithm to make DFT
  • FFTW chose the best algorithm for your machines
• Computation is split in 2 phases:
  • PLAN creation
  • Execution
• FFTW support transforms of data with arbitrary length, rank, multiplicity, and memory layout, and more....
Many different versions:

- **FFTW 2:**
  - Released in 2003
  - Well tested and used in many codes
  - Includes serial and parallel transforms for both shared and distributed memory system

- **FFTW 3:**
  - Released in February 2012
  - Includes serial and parallel transforms for both shared and distributed memory system
  - Hybrid implementation MPI-OpenMP
Multi dimensional FFT

\[ H(n_1, n_2) = \text{FFT-on-index-1} (\text{FFT-on-index-2} [h(k_1, k_2)]) \]

\[ = \text{FFT-on-index-2} (\text{FFT-on-index-1} [h(k_1, k_2)]) \]

1) For each value of \( j \) and \( k \)
Apply FFT to \( h(1...N,j,k) \)

2) For each value of \( i \) and \( k \)
Apply FFT to \( h(i,1...N,k) \)

3) For each value of \( i \) and \( j \)
Apply FFT to \( h(i,j,1...N) \)
Parallel FFT Data Distribution

Distribute data along one coordinate (e.g. $Z$)

This is know as “Slab Decomposition”
Transform along x and y

each processor transform its own sub-grid along the x and y independently of the other
Data redistribution involving x and z

The data are now distributed along x
FFT along z

each processor transform its own sub-grid
along the z dimension independently of the other
Data are redistributed, back from x to z

The 3D array now has the original layout, but each element
has been substituted with its FFT.
Some Useful Instructions
Compile a FFTW program on FERMI:

• Module Loading:
  
  \[ \text{module load fftw/3.3.3--bgq-xl--1.0} \]

• Including header:
  
  \[ -\text{I$\$FFTW\_INC} \]

• Linking:
  
  \[ -\text{L$\$FFTW\_LIB} \-lfftwf\_mpi\ -lfftw3f\_omp\ -lfftw3f\ -lm \quad (\text{single precision}) \]
  
  \[ -\text{L$\$FFTW\_LIB} \-lfftw\_mpi\ -lfftw3\_omp\ -lfftw3\ -lm \quad (\text{double precision}) \]
INCLUDING FFTW Lib:

Serial

- C:
  
  #include <fftw.h>

- FORTRAN:

  include 'fftw3.f03'

• Parallel (Pure MPI)

- C:

  #include <fftw-mpi.h>

- FORTRAN:

  include 'fftw3-mpi.f03'
Some important Remarks for FORTRAN users

• Function in C became function in FORTRAN if they have a return value, and subroutines otherwise.
• All C types are mapped via the iso_c_binning standard.
• FFTW plans are type(C_PTR) in FORTRAN.
• The ordering of FORTRAN array dimensions must be reversed when they are passed to the FFTW plan creation.
Including FFTW Lib:
- C:
  - Serial:
    ```c
    #include <fftw.h>
    ```
  - MPI:
    ```c
    include 'fftw3.f03'
    ```
- FORTRAN:
  - Serial:
    ```fortran
    include <fftw-mpi.h>
    ```
  - MPI:
    ```fortran
    include 'fftw3-mpi.f03'
    ```

MPI initialization:
- C:
  ```c
  void fftw_mpi_init(void)
  ```
- FORTRAN:
  ```fortran
  fftw_mpi_init()
  ```
Create arrays

C:
- Fixed size array:
  ```c
  fftx_complex data[n0][n1][n2]
  ```
- Dynamic array:
  ```c
  data = fftw_alloc_complex(n0*n1*n2)
  ```
- MPI dynamic arrays:
  ```c
  fftw_complex *data
  ptdiff_t alloc_local, local_no, local_no_start
  alloc_local = fftw_mpi_local_size_3d(n0, n1, n2, MPI_COMM_WORLD, &local_n0,&local_n0_start)
  data = fftw_alloc_complex(alloc_local)
  ```

FORTRAN:
- Fixed size array (simplest way):
  ```fortran
  complex(C_DOUBLE_COMPLEX), dimension(n0,n1,n2) :: data
  ```
- Dynamic array (simplest way):
  ```fortran
  complex(C_DOUBLE_COMPLEX), allocatable, dimension(:, :, :) :: data
  allocate (data(n0, n1, n2))
  ```
- Dynamic array (fastest method):
  ```fortran
  complex(C_DOUBLE_COMPLEX), pointer :: data(:, :, : )
  type(C_PTR) :: cdata
  cdata = fftw_alloc_complex(n0*n1*n2)
  call c_f_pointer(cdata, data, [n0,n1,n2])
  ```
- MPI dynamic arrays:
  ```fortran
  complex(C_DOUBLE_COMPLEX), pointer :: data(:, :, :)
  type(C_PTR) :: cdata
  integer(C_INTPTR_T) :: alloc_local, local_n2, local_n2_offset
  alloc_local = fftw_mpi_local_size_3d(n2, n1, n0, MPI_COMM_WORLD, local_n2, local_n2_offset)
  cdata = fftw_alloc_complex(alloc_local)
  call c_f_pointer(cdata, data, [n0,n1,local_n2])
  ```
Plan Creation (C2C)

1D Complex to complex DFT:
• C:
  `fftw_plan = fftw_plan_dft_1d(int nx, fftw_complex *in, fftw_complex *out, fftw_direction dir, unsigned flags)`
  
  `fftw_plan = fftw_mpi_plan_dft_1d(int nx, in, out, dir, flags)`

3D Complex to complex DFT:
• C:
  `fftw_plan = fftw_plan_dft_2d(int nx, int ny, fftw_complex *in, fftw_complex *out, fftw_direction dir, unsigned flags)`
  
  `fftw_plan = fftw_mpi_plan_dft_2d(int nx, int ny, fftw_complex *in, fftw_complex *out,MPI_COMM_WORLD, fftw_direction dir, int flags)`
  
  `fftw_plan = fftw_plan_dft_3d(int nx, int ny, int nz, fftw_complex *in, fftw_complex *out, fftw_direction dir, unsigned flags)`
  
  `fftw_plan = fftw_mpi_plan_dft_3d(int nx, int ny, int nz, fftw_complex *in, fftw_complex *out,MPI_COMM_WORLD, fftw_direction dir, int flags)`
  
• FORTRAN:
  `plan = ftw_plan_dft_1d(nx, in, out, dir, flags)`
  
  `plan = ftw_mpi_plan_dft_1d(nx, in, out, dir, flags)`
  
  `plan = ftw_plan_dft_2d(ny, nx, in, out, dir, flags)`
  
  `plan = ftw_mpi_plan_dft_2d(ny, nx, in, out, MPI_COMM_WORLD, dir, flags)`
  
  `plan = ftw_plan_dft_3d(nz, ny, nx, in, out, dir, flags)`
  
  `plan = ftw_mpi_plan_dft_3d(nz, ny, nx, in, out, MPI_COMM_WORLD, dir, flags)`
Plan Creation (R2C)

1D Complex to complex DFT:
• C:
  \[ \text{fftw\_plan} = \text{fftw\_plan\_dft\_r2c\_1d}(\text{int}\ nx, \text{double} *\text{in}, \text{fftw\_complex} *\text{out}, \text{fftw\_direction dir}, \text{unsigned flags})\]

  \[ \text{fftw\_plan} = \text{fftw\_mpi\_plan\_dft\_r2c\_1d}(\text{int}\ nx, \text{int}\ ny, \text{double} *\text{in}, \text{fftw\_complex} *\text{out}, \text{MPI\_COMM\_WORLD}, \text{fftw\_direction dir}, \text{int flags})\]

  \[ \text{FORTRAN:} \]
  \[ \text{ftw\_plan\_dft\_r2c\_1d}(nz, in, out, dir, flags) \]

3D Complex to complex DFT:
• C:
  \[ \text{fftw\_plan} = \text{fftw\_plan\_dft\_r2c\_2d}(\text{int}\ nx, \text{int}\ ny, \text{double} *\text{in}, \text{fftw\_complex} *\text{out}, \text{fftw\_direction dir}, \text{unsigned flags})\]
  \[ \text{fftw\_plan} = \text{fftw\_mpi\_plan\_dft\_r2c\_2d}(\text{int}\ nx, \text{int}\ ny, \text{int}\ nz, \text{fftw\_complex} *\text{in}, \text{fftw\_complex} *\text{out}, \text{MPI\_COMM\_WORLD}, \text{fftw\_direction dir}, \text{int flags})\]

  \[ \text{FORTRAN:} \]
  \[ \text{ftw\_plan\_dft\_r2c\_2d}(ny, nx, in, out, dir, flags) \]
  \[ \text{ftw\_mpi\_plan\_dft\_r2c\_2d}(ny, nx, in, out, \text{MPI\_COMM\_WORLD}, \text{dir}, \text{flags}) \]

3D Complex to complex DFT:
• C:
  \[ \text{fftw\_plan} = \text{fftw\_plan\_dft\_r2c\_3d}(\text{int}\ nx, \text{int}\ ny, \text{int}\ nz, \text{fftw\_complex} *\text{in}, \text{fftw\_complex} *\text{out}, \text{fftw\_direction dir}, \text{unsigned flags})\]
  \[ \text{fftw\_plan} = \text{fftw\_mpi\_plan\_dft\_r2c\_3d}(\text{int}\ nx, \text{int}\ ny, \text{int}\ nz, \text{fftw\_complex} *\text{in}, \text{fftw\_complex} *\text{out}, \text{MPI\_COMM\_WORLD}, \text{fftw\_direction dir}, \text{int flags})\]

  \[ \text{FORTRAN:} \]
  \[ \text{ftw\_plan\_dft\_r2c\_3d}(nz, ny, nx, in, out, dir, flags) \]
  \[ \text{ftw\_mpi\_plan\_dft\_r2c\_3d}(nz, ny, nx, in, out, \text{MPI\_COMM\_WORLD}, \text{dir}, \text{flags}) \]
Plan Execution

Complex to complex DFT:

- **C:**
  ```c
  void fftw_execute_dft(fftw_plan plan, fftw_complex *in, fftw_complex *out)
  void fftw_mpi_execute_dft (fftw_plan plan, fftw_complex *in, fftw_complex *out)
  ```
  ```fortran
  fftw_execute_dft (plan, in, out)
  fftw_mpi_execute_dft (plan, in, out)
  ```

Complex to complex DFT:

- **C:**
  ```c
  void fftw_execute_dft (fftw_plan plan, double *in, fftw_complex *out)
  void fftw_mpi_execute_dft (fftw_plan plan, double *in, fftw_complex *out)
  ```
  ```fortran
  fftw_execute_dft (plan, in, out)
  Fftw_mpi_execute_dft (plan, in, out)
  ```
Finalizing FFTW

Destroying PLAN:

- C:
  ```c
  void fftw_destroy_plan(fftw_plan plan)
  ```
- FORTRAN:
  ```fortran
  fftw_destroy_plan(plan)
  ```

FFTW MPI cleanup:

- C:
  ```c
  void fftw_mpi_cleanup()
  ```
- FORTRAN:
  ```fortran
  fftw_mpi_cleanup()
  ```

Deallocate data:

- C:
  ```c
  void fftw_free(fftw_complex data)
  ```
- FORTRAN:
  ```fortran
  fftw_free(data)
  ```
Some Useful Examples
program FFTW1D
  use, intrinsic :: iso_c_binding
  implicit none
  include 'fftw3.f03'
  integer(C_INTPTR_T):: L = 1024
  integer(C_INT) :: LL
  type(C_PTR) :: plan1
  complex(C_DOUBLE_COMPLEX), dimension(1024) :: idata, odata
  integer :: i
  character(len=41), parameter :: filename='serial_data.txt'
  LL = int(L,C_INT)
!! create MPI plan for in-place forward DF
  plan1 = fftw_plan_dft_1d(LL, idata, odata, FFTW_FORWARD, FFTW_ESTIMATE)
!! initialize data
  do i = 1, L
    if (i .le. (L/2)) then
      idata(i) = (1.,0.)
    else
      idata(i) = (0.,0.)
    endif
  end do
!! compute transform (as many times as desired)
  call fftw_execute_dft(plan1, idata, odata)
!! deallocate and destroy plans
  call fftw_destroy_plan(plan1)
end
program FFTW1D
use, intrinsic :: iso_c_binding
implicit none
include 'fftw3.f03'
intrinsic L = 1024
integer(L) :: LL
type(C_PTR) :: plan1
 type(C_PTR) :: p_idata, p_odata
complex(C_DOUBLE_COMPLEX), dimension(:,), pointer :: idata, odata
integer :: i
!! Allocate
LL = int(L,C_INT)
p_idata = fftw_alloc_complex(L)
p_odata = fftw_alloc_complex(L)
call c_f_pointer(p_idata, idata, (/L/))
call c_f_pointer(p_odata, odata, (/L/))
!! create MPI plan for in-place forward DF
plan1 = fftw_plan_dft_1d(LL, idata, odata, FFTW_FORWARD, FFTW_ESTIMATE)
!! initialize data
do i = 1, L
if (i.le. (L/2)) then
idata(i) = (1.,0.)
else
idata(i) = (0.,0.)
endif
end do
!! compute transform (as many times as desired)
call fftw_execute_dft(plan1, idata, odata)
!! deallocate and destroy plans
 call fftw_destroy_plan(plan1)
call fftw_free(p_idata)
call fftw_free(p_odata)
end
# include <stdlib.h>
# include <stdio.h>
# include <math.h>
# include <fftw3.h>

int main ( void )
{
    ptrdiff_t i;
    const ptrdiff_t n = 1024;
    fftw_complex *in;
    fftw_complex *out;
    fftw_plan plan_forward;
    /* Create arrays. */
    in = fftw_malloc ( sizeof ( fftw_complex ) * n );
    out = fftw_malloc ( sizeof ( fftw_complex ) * n );
    /* Initialize data */
    for ( i = 0; i < n; i++ ) {
        if ( i <= (n/2-1)) {
            in[i][0] = 1.;
            in[i][1] = 0.;
        }
        else {
            in[i][0] = 0.;
            in[i][1] = 0.;
        }
    }
    /* Create plans. */
    plan_forward = fftw_plan_dft_1d ( n, in, out, FFTW_FORWARD, FFTW_ESTIMATE );
    /* Compute transform (as many times as desired) */
    fftw_execute ( plan_forward );
    /* deallocate and destroy plans */
    fftw_destroy_plan ( plan_forward );
    fftw_free ( in );
    fftw_free ( out );
    return 0;
}
program FFT_MPI_3D
    use, intrinsic :: iso_c_binding
    implicit none
    include 'mpif.h'
    include 'fftw3-mpi.f03'

    integer(C_INTPTR_T), parameter :: L = 1024
    integer(C_INTPTR_T), parameter :: M = 1024

    type(C_PTR) :: plan, cdata
    complex(C_DOUBLE_COMPLEX), pointer :: fdata(:,:)

    integer(C_INTPTR_T) :: alloc_local, local_M, local_j_offset
    integer(C_INTPTR_T) :: i, j
    complex(C_DOUBLE_COMPLEX) :: fout
    integer :: ierr, myid, nproc

    ! Initialize
    call mpi_init(ierr)
    call MPI_COMM_SIZE(MPI_COMM_WORLD, nproc, ierr)
    call MPI_COMM_RANK(MPI_COMM_WORLD, myid, ierr)
    call fftw_mpi_init()

    ! get local data size and allocate (note dimension reversal)
    alloc_local = fftw_mpi_local_size_2d(M, L, MPI_COMM_WORLD, local_M, local_j_offset)
    cdata = fftw_alloc_complex(alloc_local)
    call c_f_pointer(cdata, fdata, [L,local_M])

    ! create MPI plan for in-place forward DFT (note dimension reversal)
    plan = fftw_mpi_plan_dft_2d(M, L, fdata, fdata, MPI_COMM_WORLD, FFTW_FORWARD, FFTW_MEASURE)
! initialize data to some function my_function(i,j)
    do j = 1, local_M
        do i = 1, L
            call initial(i, (j + local_j_offset), L, M, fout)
            fdata(i, j) = fout
        end do
    end do
! compute transform (as many times as desired)
    call fftw_mpi_execute_dft(plan, fdata, fdata)
!
! deallocate and destroy plans
    call fftw_destroy_plan(plan)
    call fftw_mpi_cleanup()
    call fftw_free(cdata)
    call mpi_finalize(ierr)
end
# include <stdlib.h>
# include <stdio.h>
# include <math.h>
# include <mpi.h>
# include <fftw3-mpi.h>

int main(int argc, char **argv)
{
    const ptrdiff_t L = 1024, M = 1024;
    fftw_plan plan;
    fftw_complex *data;
    ptrdiff_t alloc_local, local_L, local_L_start, i, j, ii;
    double xx, yy, rr, r2, t0, t1, t2, t3, tplan, texec;
    const double amp = 0.25;
    /* Initialize */
    MPI_Init(&argc, &argv);
    fftw_mpi_init();

    /* get local data size and allocate */
    alloc_local = fftw_mpi_local_size_2d(L, M, MPI_COMM_WORLD, &local_L, &local_L_start);
    data = fftw_alloc_complex(alloc_local);
    /* create plan for in-place forward DFT */
    plan = fftw_mpi_plan_dft_2d(L, M, data, data, MPI_COMM_WORLD, FFTW_FORWARD, FFTW_ESTIMATE);
/* initialize data to some function my_function(x,y) */
/* ..........*/
/* compute transforms, in-place, as many times as desired */
    fftw_execute(plan);
/* deallocate and destroy plans */
    fftw_destroy_plan(plan);
    fftw_mpi_cleanup();
    fftw_free ( data );
    MPI_Finalize();
}
LIMITATIONS
Slab Decomposition
2D Domain Decomposition
A FORTRAN Library that use 2D Domain Decomposition
Compile a FFTW program on FERMI:

- **Module Loading:**
  
  module load profile/advanced

  module load autoload 2Decomp_fft/1.5.847--bgq-xl--1.0

- **Including header:**

  -I$FFTW_INC  -I$DECOMP_2D_FFT_LIB

- **Linking:**

  -L$DECOMP_2D_FFT_LIB  -L$FFTW_LIB  -l2decomp_fft  -lfftw3_mpi  -lfftw3f  -lfftw3  -lm
Including 2Decomp&FFT Lib:
use decomp_2d
use decomp_2d_fft

Initial declarations:
integer, parameter :: p_row = ..
integer, parameter :: p_col = ...
complex(mytype), allocatable, dimension(:, :, :) :: in, out
Please note that: p_row*p_col = N_core

Initialization:
call decomp_2d_init(n0,n1,n2,p_row,p_col)
call decomp_2d_fft_init

Allocation:
allocate (in(xstart(1):xend(1),xstart(2):xend(2),xstart(3):xend(3)))
allocate (out(zstart(1):zend(1),zstart(2):zend(2),zstart(3):zend(3)))

Execution:
call decomp_2d_fft_3d(in, out, DECOMP_2D_FFT_FORWARD)

Finalizing:
call decomp_2d_fft_finalize
call decomp_2d_finalize
An example
PROGRAM FFT_3D_2Decomp_MPI
  use mpi
  use, intrinsic :: iso_c_binding
  use decomp_2d
  use decomp_2d_fft
  implicit none
  integer, parameter :: L = 128
  integer, parameter :: M = 128
  integer, parameter :: N = 128
  integer, parameter :: p_row = 16
  integer, parameter :: p_col = 16
  integer :: nx, ny, nz
  complex(mytype), allocatable, dimension(:,:, :) :: in, out
  complex(mytype) :: fout
  integer :: ierror, i,j,k, numproc, mype
  integer, dimension(3) :: sizex, sizez

! ===== Initialize
  call MPI_INIT(ierr)
  call decomp_2d_init(L,M,N,p_row,p_col)
  call decomp_2d_fft_init
allocate (in(xstart(1):xend(1),xstart(2):xend(2),xstart(3):xend(3)))
allocate (out(zstart(1):zend(1),zstart(2):zend(2),zstart(3):zend(3)))

! ===== each processor gets its local portion of global data =====
do k=xstart(3),xend(3)
do j=xstart(2),xend(2)
do i=xstart(1),xend(1)
call initial(i, j, k, L, M, N, fout)
in(i,j,k) = fout
end do
end do
end do

! ===== 3D forward FFT =====
call decomp_2d_fft_3d(in, out, DECOMP_2D_FFT_FORWARD)

! =====================
call decomp_2d_fft_finalize
call decomp_2d_finalize
deallocate(in,out)
call MPI_FINALIZE(ierr)
end
Some results
Links:

**FFTW Homepage**: [http://www.fftw.org/](http://www.fftw.org/)

**Download FFTW-3**: [http://www.fftw.org/fftw-3.3.3.tar.gz](http://www.fftw.org/fftw-3.3.3.tar.gz)


**2Decomp&FFT homepage**: [http://www.2decomp.org/](http://www.2decomp.org/)

**Download 2Decomp&FFT**: [http://www.2decomp.org/download/2decomp_fft-1.5.847.tar.gz](http://www.2decomp.org/download/2decomp_fft-1.5.847.tar.gz)
Thank You